

PRIMITIVE GEODESIC LENGTHS AND (ALMOST) ARITHMETIC PROGRESSIONS

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Abstract: In this article we investigate when the set of primitive geodesic lengths on a Riemannian manifold have arbitrarily long arithmetic progressions. We prove that in the space of negatively curved metrics, a metric having such arithmetic progressions is quite rare. We introduce almost arithmetic progressions, a coarsification of arithmetic progressions, and prove that every negatively curved, closed Riemannian manifold has arbitrarily long almost arithmetic progressions in its primitive length spectrum. Concerning genuine arithmetic progressions, we prove that every non-compact, locally symmetric, arithmetic manifold has arbitrarily long arithmetic progressions in its primitive length spectrum. We end with a conjectural characterization of arithmeticity in terms of arithmetic progressions in the primitive length spectrum. We also suggest an approach to a well known spectral rigidity problem based on the scarcity of manifolds with arithmetic progressions.

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Key words: Primitive geodesic, almost arithmetic progression, specification property, locally symmetric space, arithmetic manifold, modular surface.