SEMINARI D'ANÀLISI UAB-UB **Curs 2013–2014**

Dilluns 14 d'octubre del 2013, 15:00h

Aula Petita (CRM).

On the ∞-laplacian, the ∞-mean value property and unique continuation

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ABSTRACT:

In the same way the usual laplacian arises from minimization of the L^2 norm in a Dirichlet boundary value problem, minimization of the L^∞ norm originates the so called ∞-laplacian Δ_∞, a highly degenerated nonlinear operator which, for sufficiently smooth u, is given by

$$\Delta_\infty u = \sum u_{x_i}\, u_{x_j}\, u_{x_i x_j}$$

Its solutions (in an appropiate weak sense) are called ∞-harmonic functions. In the talk we will review some situations where the ∞-laplacian comes up and also some facts and open problems, with special emphasis on its connections to mean value properties and questions of unique continuation.

While harmonic functions are characterized by the usual mean value property, it turns out that the ∞-laplacian is closely related to the following nonlinear mean value property (∞-MVP):

$$u(x) = \frac{1}{2}\left(\sup_B u + \inf_B u\right)$$

(where B is a ball centered at x). Those continuous functions in a domain $\Omega \subset \mathbb{R}^n$ satisfying the ∞-MVP for balls of small enough radius form a (strict) subclass of the ∞-harmonic functions in Ω. We will see that if $u \equiv 0$ on any proper $\Omega' \subset \Omega$ then $u \equiv 0$ on Ω. The corresponding problem for the whole class of ∞-harmonic functions is open.