

# ON THE CURRENT STATE OF THE ALMOST PERIODIC FACTORIZATION PROBLEM: A SURVEY

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Factorization of matrix functions (that is, their representation as products of multiples analytic inside and outside the given closed curve) arises naturally in many applications, including those to convolution type equations on a half-line (the classical Wiener-Hopf method). As it happens, the equations on finite intervals also can be treated via the factorization method. The resulting matrix functions, however, are of oscillating type, which has not been treated until recently. The general case can be boiled down to the situation when the matrix is almost periodic, that is, its elements belong to the algebra generated by  $\exp(iax)$  with real values of the parameter  $a$ . We will discuss the current state of the factorization problem for such matrices. A special attention will be paid to a (seemingly narrow) case of 2-by-2 triangular matrix functions, but even for them the factorability properties remain a mystery, in striking difference with both the scalar almost periodic case and the classical Wiener-Hopf setting.