Positivity in operator theory: Domination problems

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ABSTRACT:

In the framework of Banach lattice theory a central topic is the study of relations between the order and Banach space structure.

A particular case of this question is the following: Given an operator ideal $\mathcal{I}$ (or a more general class of operators) we are interested in the following problems:

- **Domination problem**: Let $0 \leq S \leq T : E \to F$. Under which conditions on the Banach lattices E and F does $T \in \mathcal{I}$ imply $S \in \mathcal{I}$?

- **Power problem**: Does there exist $n \in \mathbb{N}$ such that if $0 \leq S \leq T : E \to F$ and $T \in \mathcal{I}$ then $S^n \in \mathcal{I}$?

For instance, a classical result due to P. Dodds and D. Fremlin asserts that for the ideal of compact operators $\mathcal{K}$, the domination problem has a positive answer when E^* and F are order continuous. Similarly, for the power problem, S^3 is always a compact operator provided that $0 \leq S \leq T : E \to E$ and T is compact. We will survey some old and new results for several operator ideals.