SEMINARI D'ANÀLISI UAB-UB **Curs 2012–2013**

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Aula A1 (CRM).

On the Vortex Filament Equation for a Regular Polygon

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ABSTRACT: Given a curve $\mathbf{X}_0 : \mathbb{R} \longrightarrow \mathbb{R}^3$, we consider the geometric flow $\mathbf{X}_t = c\mathbf{b}$, where c is the curvature and $\mathbf{b}$ the binormal component of the Frenet–Serret formulae. It can be expressed as

$$\mathbf{X}_t = \mathbf{X}_s \wedge \mathbf{X}_{ss}, \tag{0.1}$$

where $\wedge$ is the usual cross-product, and s denotes the arc-length parameterization. This flow, known as the Localized Induction Approximation (LIA) or the Vortex Filament Equation, appeared for the first time in 1906 [2], and was rederived in 1965 by Arms and Hama [1] as an approximation of the dynamics of a vortex filament under the Euler equations. Some of its explicit solutions are the line, circle, and helix. Since the tangent vector $\mathbf{T} = \mathbf{X}_s$ remains with constant length, we can assume that it takes values on the unit sphere. Differentiating (0.1), we get the so-called Schrödinger map equation on the sphere:

$$\mathbf{T}_t = \mathbf{T} \wedge \mathbf{T}_{ss}. \tag{0.2}$$

In [3], we have considered the evolution of (0.1) and (0.2), taking a planar regular polygon of M sides as $\mathbf{X}(s, 0)$. Assuming uniqueness and bearing in mind the invariances and symmetries of (0.1) and (0.2), we are able to fully characterize, by algebraic means, $\mathbf{X}(s, t)$ and $\mathbf{T}(s, t)$, at rational multiples of $t = 2\pi/M^2$. We show that the values at those points are intimately related to the generalized quadratic Gauß sums:

$$G(a, b, c) = \sum_{l=0}^{c-1} e^{2\pi i(al^2+bl)/c}. \tag{0.3}$$

We also mention some fractality phenomena appearing during the evolution of $\mathbf{X}$ and $\mathbf{T}$. All the results are completely supported by numerical simulations.

References

[1] R. J. Arms, F. R. Hama, *Localized-Induction Concept on a Curved Vortex and Motion of an Elliptic Vortex Ring*, Phys. Fluids, 8(4):553–559, 1965.

[2] L. S. Da Rios, *On the motion of an unbounded fluid with a vortex filament of any shape*, Rend. Circ. Mat. Palermo, 22:117–135, 1906. In Italian.

[3] F. de la Hoz, L. Vega, *Vortex filament equation for a regular polygon*, preprint, 2013.