

Bilinear Forms on the Dirichlet Space

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ABSTRACT. (Joint work with R. Rochberg, E. Sawyer, B. Wick). Let \mathcal{D} be the classical Dirichlet space, the Hilbert space of holomorphic functions on the disk. Given a holomorphic symbol function b we define the associated Hankel type bilinear form, initially for polynomials f and g , by $T_b(f, g) := \langle fg, b \rangle_{\mathcal{D}}$, where we are looking at the inner product in the space \mathcal{D} . We let the norm of T_b denote its norm as a bilinear map from $\mathcal{D} \times \mathcal{D}$ to the complex numbers. We say a function b is in the space \mathcal{X} if the measure $d\mu_b := |b'(z)|^2 dA$ is a Carleson measure for \mathcal{D} , and we norm \mathcal{X} by

$$\|b\|_{\mathcal{X}} := |b(0)| + \left\| |b'(z)|^2 dA \right\|_{CM(\mathcal{D})}^{1/2}.$$

Our main result is that T_b is bounded if and only if $b \in \mathcal{X}$ and

$$\|T_b\|_{\mathcal{D} \times \mathcal{D}} \approx \|b\|_{\mathcal{X}}.$$