SEMINARI D'ANÀLISI UAB-UB **Curs 2013–2014**

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Aula T2 (UB).

On Gagliardo-Nirenberg type estimates

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ABSTRACT: The Sobolev space $W^1L^p(I^n)$, $1 \leq p \leq \infty$, consists of all functions in $L^p(I^n)$ whose first-order distributional derivatives also belong to $L^p(I^n)$. The classical Sobolev embedding theorem claims:

$$W^1L^p(I^n) \hookrightarrow L^{pn/(n-p)}(I^n), \quad 1 \leq p < n.$$

Sobolev proved this embedding for $p > 1$, but his method, based on integral representations, did not work when $p = 1$. That case was settled affirmatively by Gagliardo and Nirenberg, who first observed:

$$W^1L^1(I^n) \hookrightarrow \mathcal{R}(L^1, L^\infty), \tag{0.1}$$

where $\mathcal{R}(L^1, L^\infty)$ is a mixed norm space, and then, using an iterated form of Hölder's inequality, completed the proof. Our main goal in this work is to study the embedding (0.1) for more general rearrangement invariant (r.i.) spaces. In particular we concentrate on seeking the optimal domains and the optimal ranges for these embeddings between r.i. spaces and mixed norm spaces. As a consequence, we prove that the classical estimate for the standard Sobolev space W^1L^p by Poornima and Peetre ($1 \leq p < n$), and by Hansson, Brézis, Wainger and Maz'ya ($p = n$) can be improved considering mixed norms as targets spaces.

This work is part of my PhD thesis, supervised by Javier Soria (University of Barcelona).