OPERADIC CATEGORIES, DECALAGE, AND DECOMPOSITION SPACES

JOACHIM KOCK

Batanin and Markl introduced the notion of operadic category as a machinery used to prove the duoidal Deligne conjecture. Operadic categories C have algebras called C-operads. For example, ordinary symmetric operads are algebras for the terminal operadic category F, the category of finite sets, in the spirit of viewpoints tracing back to Day-Street and Barwick. An operadic category is a small category equipped with a notion of cardinality, a notion of abstract fibre, and a choice of local terminal objects, and this data is subject to 9 axioms, mimicking the way these notions behave in F. At CT 2014, Lack gave a more conceptual reinterpretation of the notion of operadic category in terms of skew monoidal categories. In this talk I will explain a different reinterpretation of the notion, based on the decalage comonad D. Operadic categories are exhibited as algebras for a monad on the category of D-coalgebras sliced over F, and that monad is itself induced by D. One benefit of the approach is that it reveals relationships with decomposition spaces (2-Segal spaces).

This is joint work with Richard Garner and Mark Weber.